THE LEBANESE AMERICAN UNIVERSITY

Fall 2016 - MTH305

EXAM 1

October 14, 2016

Name: ID:

This exam comprises 10 problems. Answer the questions in the space provided for each problem. If more space is needed, use the back of the page. Make sure to justify all your answers.

Duration: 55 minutes

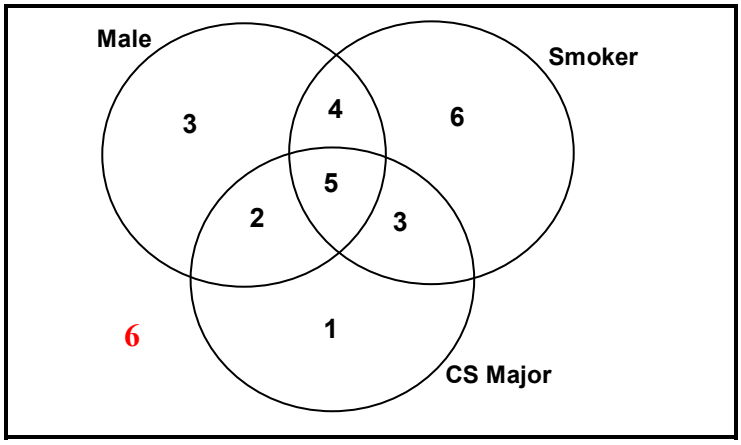
1. (10 pts.) In how many ways can the following be done:

    a. A computer password (made of 5 distinct alphanumeric characters) such that the first character is to be an odd digit, and the second and third characters are to be vowels. The 4^{th} and 5th characters can be non-zero digits or letters. (The password is written from left to right). Hint: There are 26 letters of which 6 are vowels.

    b. Three maple trees, two pine trees and four oak trees to be planted along a property line if one cannot distinguish between the maple and pine trees.

2. (10 pts.) In a class of 80 students, 55 are "Smoker or Male" and 50 are "Female or Smoker". Calculate the number of smokers.

3. (15 pts.) The Venn-diagram below shows the distribution of MTH375 students by three different categories: Gender, Major and Smoking habit.



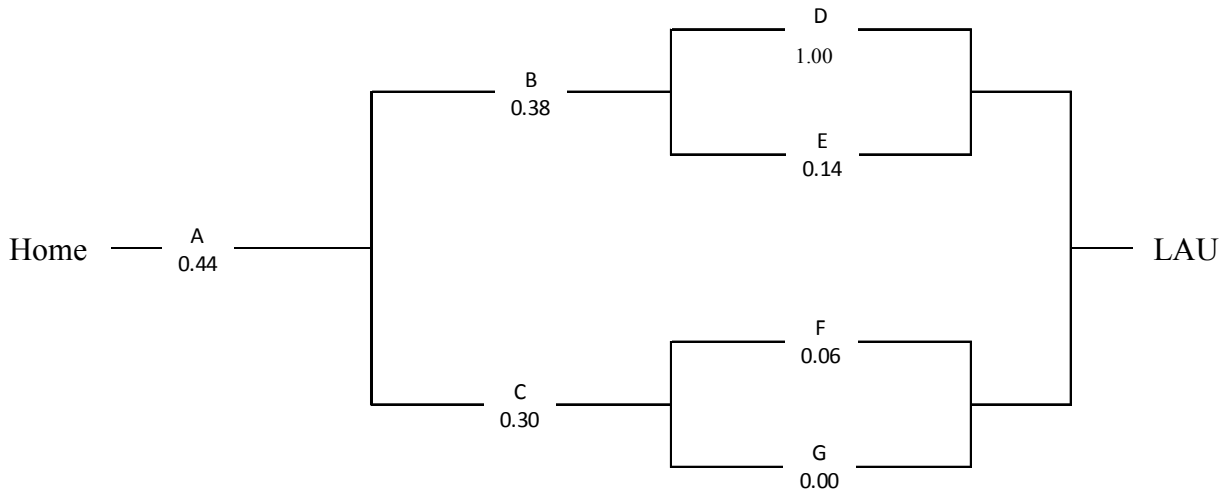
A student was randomly selected calculate the following:

a. The probability that the selected student is CS major or non-smoker.

b. The probability that selected student is CS major given that he is male non-smoker.

c. The probability that the selected student is female or smoker or non CS major.

4. (10 pts.) A student is going from his home to LAU early in the morning. The following diagram shows the possible routes the student can take to arrive to LAU. The student cannot go through if there are demonstrations on his way. The probability of **NOT** having demonstrations on each of points A, B, C, D, E, F and G are given below.
   a. Calculate the probability that the student will not arrive to LAU assuming that demonstrations at different locations are independent.



   b. In problem (1) above, given that the student didn't arrive to LAU, what's the probability that there was a strike at point F?

5. (10 pts.) An auto insurance company insures drivers of all ages. Based on available data, 20% of the insured drivers made accidents of which 15% are less than 20 years old. In addition, of all the insured drivers, 48% are above 20 years **and** made no accidents.
   Calculate the probability that a young driver (less than 20 years) will make an accident. Hint: draw a tree diagram

6. (5 pts.) Let X and Y be continuous random variables with joint density function:

$$f(x,y)=\begin{cases} kxy & 1\le x\le 2, \quad x\le y\le 2x \\ 0 & elsewhere \end{cases}$$

   Find *k*

7. (10 pts.) Let X and Y be continuous random variables with joint density function:

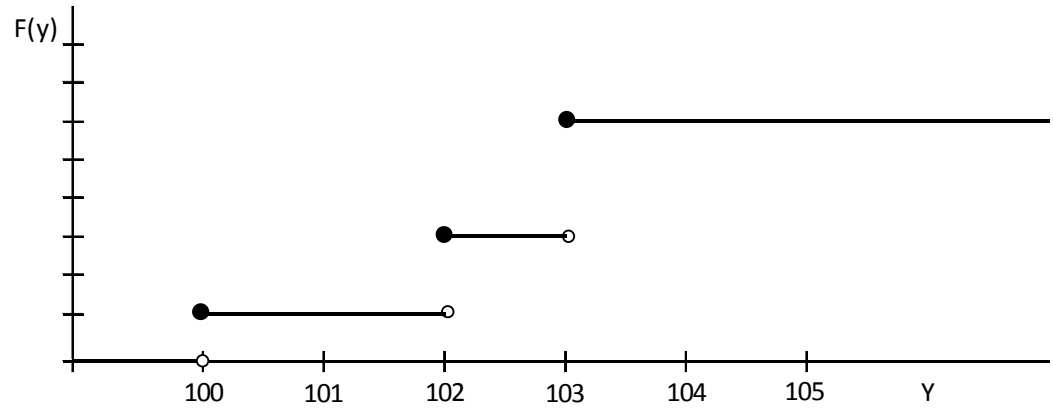
$$f(x,y)=\begin{cases}\frac{8}{9}xy & 1<x<y<2\\ 0 & elsewhere\end{cases}$$

   a. Find the marginal distribution function of X

   b. Calculate $P(2X<3)$

8. (10 pts.) An IT company has 5,000 employees. Of these employees, 1500 are young, 2300 are male, and 3500 are married. In addition, 660 of the employees are young males, 1505 are married males, and 700 are young married persons. Finally, 300 of the employees are young, male and married.
   Calculate the number of the company's employees who are young, female, and unmarried. Hint: draw a Venn-diagram.

9. (10 pts.) Given the following graph of the cumulative distribution function of random variable Y.



Calculate P(Y=103)

10. (10 pts.) The temperature in degree Celsius in a city in Northern France is modeled by random variable X with the following probability density function:

$$f(x)=\begin{cases}\frac{2}{225}(15-x), & 0<x<15\\ 0 & \textit{elsewhere}\end{cases}$$

Given that the temperature exceeds 8°C, calculate the probability that it does not exceed 12°C